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Kinetic (PIC) simulations for a plane probe in a collisional plasma

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Abstract

In Langmuir-probe diagnostics of fusion plasmas, collisions between charged and neutral particles are often neglected. We present a simple analytical theory of the plane probe including these collisions. Our analytical results are confirmed by high-resolution particle-in-cell (PIC) simulations covering the whole range of the Volt–Ampère characteristic.

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1. Introduction

The Langmuir probe is a powerful tool for plasma diagnostics. However, the related theory in its classical approach takes into account just a simplified model of the collisions between charged and neutral particles which are usually present in fusion edge plasmas. Here we develop a simple plane-probe theory accounting for these charged-neutral collisions more accurately.

As in the classical approach, our model is based on ion fluid dynamics and electrons with a cut-off velocity distribution. Thus, some kinetic effects cannot be included self-consistently. In order to clarify these effects and also to cross-check our results, we have made corresponding kinetic (particle-in-cell, PIC) simulations.

2. Theory of the negatively biased, unmagnetized plane collisional probe

In front of a conducting wall or probe, a narrow plasma layer, called the plasma-wall transition (PWT) layer, is formed. Generally, this layer consists of a nonneutral Debye sheath (DS) and a quasineutral presheath which can be determined by ion-neutral collisions, ionization,

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divergent geometry, and/or a magnetic field oblique to the probe surface [1]. Here we assume plane geometry and a vanishing or normal magnetic field, so that we have a collisional presheath (CP) determined by ion-neutral collisions and electron-impact ionization.

A classical plane-probe characteristic consists of three current regimes: the electron-saturation–current regime (A), the retarding-field regime (B), and the ion-saturation–current regime (C). The electrons are assumed to be Boltzmann-distributed, thus the electron current to the probe scales as the thermal velocity times the Boltzmann factor [2]. For the ion current one uses the Bohm condition [3] and the assumption that the total potential drop across the presheath is $kT_e/2$. Then the total current to the probe is given by [2]

$$I = eAn_\infty \left(\frac{kT_e}{m_i}\right)^{1/2} \left[\left(\frac{m_i}{2\pi m_e}\right)^{1/2} \exp\left(\frac{eU_{tot}}{kT_e}\right) - \exp\left(-\frac{1}{2}\right)\right], \tag{1}$$

where A is the area of the probe, n_∞ is the bulk-plasma density, T_e is the electron temperature, U_{tot} is the total potential drop between the probe and the bulk plasma, and m_i and m_e are the ion and electron masses, respectively.

It is important to note that (i) this expression is not valid for the electron-saturation–current regime, (ii) the electron velocity distribution function has a cut-off due to superthermal electrons absorbed at the probe, so that the corresponding density cannot be given just by the Boltzmann factor, (iii) the potential drop across the CP is a function of collisionality and in general is not $kT_e/2$, and (iv) the ion dynamics inside the CP does not necessarily obey a global polytropic law with a constant coefficient γ as is often assumed [1].

In the present work we try to improve the classical theory outlined above for negatively biased probes, i.e., for the retarding-field and ion-saturation–current regimes (B and C). The probe surface is located on the left-hand side of the PWT region considered by us.

In order to derive the expression for the ion current we consider the ion continuity and momentum-conservation equations for an unmagnetized plasma [4]

$$\frac{\partial}{\partial x}(n_i u_i) = n_i \nu_i, \tag{2}$$

$$u_i \frac{\partial u_i}{\partial x} + \frac{e}{m_i}\frac{\partial \phi}{\partial x} = -\frac{k}{n_i m_i}\nabla(n_i T_i) - (\nu_i + \nu_{mt}) u_i, \tag{3}$$

where ν_i and ν_{mt} are the electron impact ionization frequency and the ion-neutral momentum transfer collision frequency, respectively.

We introduce the electron screening temperature [1]

$$\frac{kT_e^*}{n_e}\frac{dn_e}{dx} = e\frac{d\phi}{dx} \tag{4}$$

and the local polytropic coefficient

$$\gamma = 1 + \frac{n_i}{T_i}\left(\frac{dn_i}{dx}\right)^{-1}\frac{dT_i}{dx} \tag{5}$$

corresponding to the local polytropic law $\lfloor p_i(\delta V)^\gamma \rfloor_{x+dx} = \lfloor p_i(\delta V)^\gamma \rfloor_x$, where δV is the volume of a given ion fluid element moving from position x to position $x + dx$. In general, γ depends on the position x and will not necessarily be constant as is usually assumed in classical presheath theory [1].

Using the quasineutrality condition inside the CP, we obtain from Eqs. (2) and (3)

$$\frac{du_i}{dx}\left(\frac{c^2}{u_i^2} - 1\right) = (\nu_i + \nu_{mt})\left(1 + \frac{\nu_i c^2}{(\nu_i + \nu_{mt}) u_i^2}\right) \tag{6}$$

and from Eqs. (2)–(5)

$$d(\ln n_i) = -\frac{2\nu_i + \nu_{mt}}{\nu_i + \nu_{mt}} \frac{1}{u_i\left(1 + \frac{c^2}{u_i^2}\frac{\nu_i}{\nu_i + \nu_{mt}}\right)} du_i, \tag{7}$$

where

$$c(x) = \sqrt{\frac{k(T_e^* + \gamma T_i)}{m_i}} \tag{8}$$

is the ion-sound velocity. The Debye-sheath entrance (SE) is associated with the sheath singularity condition, $u_{i\,SE} = c_{SE}$. By this condition (which usually coincides with the Bohm condition for sheath formation [1]), all gradients become infinite at the SE.

Now we neglect the variation of c inside the CP and integrate the expression (7) across the CP to obtain

$$n_i^{SE} = n_\infty \frac{2\nu_i + \nu_{mt}}{\nu_i + \left(\frac{u_{i0}}{c_{SE}}\right)^2 (\nu_i + \nu_{mt})} \exp\left(-\frac{\nu_i + \nu_{mt}/2}{\nu_i + \nu_{mt}}\right), \tag{9}$$

where n_i^{SE} is the ion density at the SE and u_{i0} is the average ion velocity at the collisional-presheath entrance (PE). Thus the ion current to the probe (which is the same as the ion current at the SE) is given as

$$I_{isat} = eAn_i^{SE} c_{SE} \tag{10}$$

with n_i^{SE} from Eq. (9).

Let us next derive the expression for the electron current. Here, in contrast to the classical approach, we take into account a cut-off distribution function, which is due the absorption of superthermal electrons by the probe. The electron-neutral collision mean free path is much larger than the size of the CP, which is of the order of the ion mean free path. Hence, the electrons can be considered to be collisionless, and assuming that they are thermalized in the bulk plasma their distribution function inside the PWT is well approximated by the cut-off Maxwellian

$$n_e(x) = n_\infty \frac{1+\mathrm{erf}\sqrt{\frac{e(\phi(x)-V_p)}{kT_e^{pl}}}}{1+\mathrm{erf}\sqrt{\frac{e(V_{pl}-V_p)}{kT_e^{pl}}}} \exp\left(\frac{e\phi(x)}{kT_e^{pl}}\right). \quad (11)$$

Here, V_p (<0) and V_{pl} (=0) are the potential values at the probe surface and in the bulk plasma, respectively, and $\phi(x) < 0$ everywhere inside the CP; T_e^{pl} is the electron temperature in the bulk plasma, which is related to the effective electron temperature T_e^{eff} by

$$T_e^{eff}(x) = \frac{1}{k}\left\langle m_e(v-\langle v\rangle)^2\right\rangle \leqslant T_e^{pl}. \quad (12)$$

Using Eq. (11) we can write the screening temperature as $T_e^*(x)$

$$= T_e^{pl}\left[1+\frac{1}{\sqrt{\pi}}\frac{\exp\left(-\frac{e(\phi(x)-V_p)}{kT_e^{pl}}\right)}{\sqrt{\frac{e(\phi(x)-V_p)}{kT_e^{pl}}}\left(1+\mathrm{erf}\sqrt{\frac{e(\phi(x)-V_p)}{kT_e^{pl}}}\right)}\right]^{-1} \quad (13)$$

and electron current as

$$I_e = eAn_\infty \frac{v_{e,th}}{\sqrt{2\pi}} \frac{2}{1+\mathrm{erf}\sqrt{\frac{e\mid U_{tot}\mid}{kT_e^{pl}}}} \exp\left(\frac{eU_{tot}}{kT_e^{pl}}\right). \quad (14)$$

Here $v_{e,th} = \sqrt{kT_e^{pl}/m_e}$ and $U_{tot} = V_{pl} - V_p$ are the thermal electron velocity and the total potential drop between the probe and the bulk plasma, respectively.

Eqs. (10) and (14) result in a total current to the probe (for the regions B and C of the probe characteristic)

$$I = eAn_\infty\left[\frac{v_{e,th}}{\sqrt{2\pi}} \frac{2}{1+\mathrm{erf}\sqrt{\frac{e\mid U_{tot}\mid}{kT_e^{pl}}}} \exp\left(\frac{eU_{tot}}{kT_e^{pl}}\right) - c_{SE}\cdot\left(\frac{2v_i+v_{mt}}{v_i+\left(\frac{u_{i0}}{c_{SE}}\right)^2(v_i+v_{mt})}\right)^{-\frac{v_i+v_{mt}/2}{v_v+v_{mt}}}\right] \quad (15)$$

with c_{SE} from Eq. (8) evaluated at the SE. Note that the current is positive if more electrons than ions are coming to the probe.

Using this analytical model for electrons and ions (to be referred to as the 'AEI' model), we construct regions B and C of the Volt–Ampère characteristic. In Fig. 1 we show the difference between the classical theory and our AEI model. To estimate the ion current we determined the polytropic coefficient from the simulation results (Fig. 2). In addition, to check this result, we simplified our AEI model using the assumption that, for all the points in the B regime, the ion current is constant and equal to the ion saturation current. In this case, the electron screening temperature T_e^* is identical with the bulk-plasma electron temperature T_e^{pl}, the polytropic coefficient becomes $\gamma = 1$ near the SE and the ion temperature there equals approximately half its value at the injection point. With these considerations, the differences between this simplified model ('AEI simplified') and the AEI model are shown in Fig. 1.

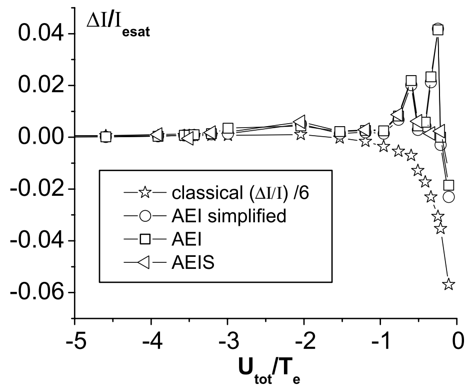

Fig. 1. Differences in normalized total currents from different models vs. normalized potential. Here, $I_{esut} = eAn_\infty v_{e,th}/\sqrt{2\pi}$ is the electron saturation current, ΔI is the differences between the currents from PIC and different analytical models. 'AEI', 'AEIS' and 'AEI simplified' correspond to the fully analytical model (Eq. (15)), to the model with the analytic electron and simulated ion currents, and to the simplified model with $\gamma = 1$, $T_e^{pl} = T_e^*$, respectively.

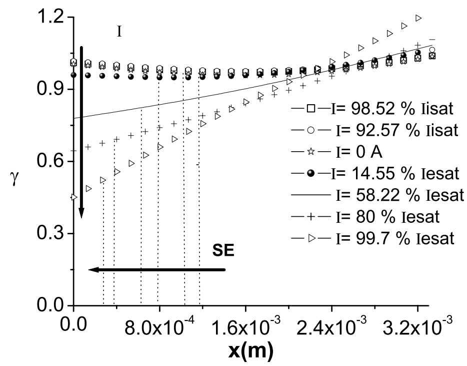

Fig. 2. Polytropic coefficient profile from PIC simulations.

3. PIC Simulations

Related to the considerations of Section 2, PIC simulations will be useful in providing information about the profiles of the plasma parameters, the polytropic coefficient, and the screening temperature throughout the PWT region considered. We perform 1d3v (one spatial and three velocity dimensions) PIC simulations using the code BIT1 [5], which was developed at Innsbruck University on the basis of the XPDP1 code from UC Berkeley [6]. Electrons and singly charged ions with half Maxwellian distribution functions are injected from the right into the system, which is empty initially. The probe on the left is assumed to be perfectly absorbing. For the plasma parameters we have chosen values relevant to a Tokamak deuterium plasma. The right-hand boundary is considered to be the PE, where the injection plasma parameters are chosen as $n_i = n_e = 4.3 \times 10^{17}\ m^{-3}$; $T_i = T_e = 20$ eV. Atomic deuterium constitutes a fixed background with density $n_n = 2 \times 10^{19}\ m^{-3}$ and temperature $T_n = 0.001$ eV. At present, the BIT1 code does not follow neutrals but rather assumes fixed neutral density and temperature profiles. All charged-neutral particle collisions for deuterium (relevant to the SOL) are implemented, namelyelastic ($D + e \rightarrow D + e$), excitation ($D + e \rightarrow D^* + e$), ionization ($D + e \rightarrow D^+ + 2e$), elastic ($D + D^+ \rightarrow D + D^+$), and charge exchange ($D + D^+ \rightarrow D^+ + D$) [7,8].

In Fig. 3 We show the simulated probe characteristic for the regions B and C. Using Eq. (5), with the ion temperature and density profiles determined from the simulation, we calculate γ for different current regimes. In Fig. 2 are shown the γ values at the SE for different points in the probe characteristic. Inserting in Eq. (8) the plasma parameter profiles obtained from the simulation, we found the sound velocity profiles for regions B and C. Comparing these profiles with the average ion velocity profiles, the intersection point will be the SE. In Fig. 4 we plot, as an example, these two profiles for the case $I = 98\% I_{isat}$. The position of the SE for different points in the probe characteristic and the corresponding ion acoustic velocity is plotted in Fig. 5. Both c_{SE} and the sheath length are seen to decrease if the total probe current increases. This is caused by the decrease in T_e^* (due to the increased cut-off in the electron distribution) and γ. The smallest simulated potential drop corresponds to $U_{tot} = -2$ V (the last point in Fig. 3).

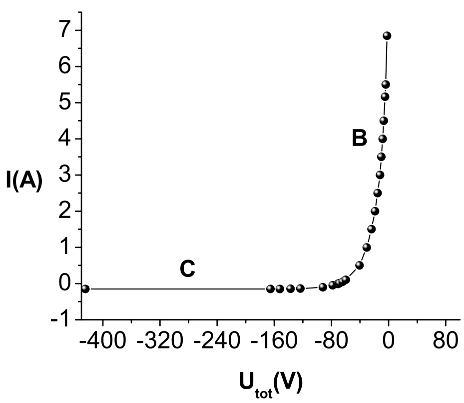

Fig. 3. Volt–Ampère characteristic from simulation.

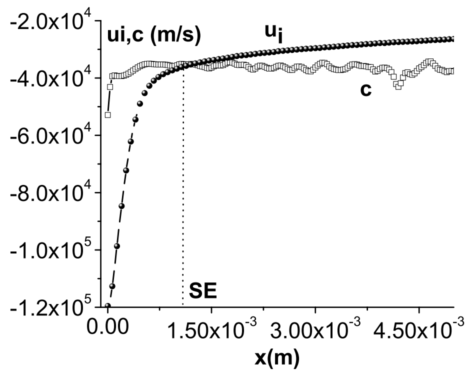

Fig. 4. The sound velocity and the ion average velocity profiles from PIC simulations for $I = 98\% I_{isat}$.

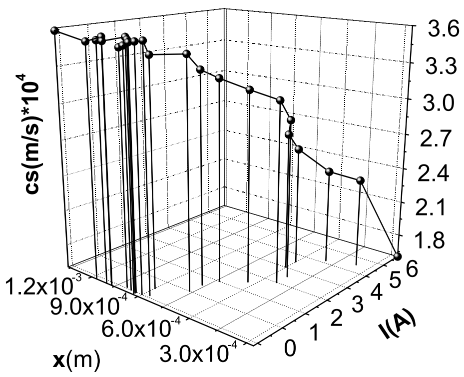

Fig. 5. The ion sound velocity at the SE for different current regimes.

With this analysis for the ions and the analytical formula (14) for the electrons, the difference between the total current calculated applying this model (AEIS) and the one obtained from simulations is presented in Fig. 1.

4. Conclusions

We have made fully self-consistent simulations for a plane probe immersed in a collisional deuterium plasma and an improved the corresponding analytical model. Our analytical model shows good agreement with the collisional-plasma simulation. We have shown that (a) the ion current in the B and C regions can be well approximated by Eq. (10), and (b) the cut-off in the electron distribution has to be necessarily taken into account for regimes near electron- current saturation.

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